SOME PROPERTIES OF VECTOR SPACES

(1) 0_V is unique and the addition inverse $-u$ is unique for each $u \in V$.

(2) $0 \odot u = 0_V$.

(3) $c \odot 0_V = 0_V$.

(4) $c \odot u = 0_V$, then either $c = 0$ or $u = 0_V$.

(5) $(-1) \odot u = -u$.

I will provide the proofs for (1) and (5).

Proof. (1) Assume there are zero vectors 0_V and $\tilde{0}_V$, then by condition (3) in the definition of vector spaces:

$$0_V + \tilde{0}_V = \tilde{0}_V.$$

However $\tilde{0}_V$ is also a zero vector, so

$$0_V + \tilde{0}_V = 0_V = \tilde{0}_V.$$

So the zero vector is unique. The proof for the second part is essentially the same.

(5) By conditions (6) and (8) in the definition of vector spaces:

$$(-1) \odot u \oplus u = (-1) \odot u \oplus (1 \odot u) = (-1 + 1) \odot u = 0 \odot u = 0_V.$$

The last equality follows from (2). Since by (1) the addition inverse of u is unique, we infer that $(-1) \odot u = -u$. □